## CURRICULUM

FOR

## TWO-YEAR ASSOCIATE DEGREE /

BS (4-YEARS) IN
MATHEMATICS
2023 AND ONWARDS


DEPARTMENT OF MATHEMATICS UNIVERSITY OF MALAKAND CHAKDARA DIR (L), KPK, PAKISTAN


## Main Structure

| S. No. | Categories | Number of <br> Courses | Credit Hours |
| :---: | :--- | :---: | :---: |
| 1 | General Education Cluster | 12 | 30 |
| 2 | Allied Courses | 07 | 21 |
| 3 | Major Courses | 25 | 79 |
| 4 | Research project | 01 | 03 |
| Total |  |  |  |

The BS Mathematics Scheme of Studies: Layout/ Framework

| General Education Cluster |  | Allied Courses |  |
| :---: | :---: | :---: | :---: |
| 12 courses |  | 7 courses |  |
| 30 Credit hours |  | 21 Credit hours |  |
| Title | Credit <br> hours | Title | Credit <br> hours |
| 1. Islamic Studies/ Ethics | 2 | 1. Computer Programing | 3(2+1) |
| 2. Ideology and Constitution of Pakistan | 2 | 2. Physics-I | 3 |
| 3. Functional English | 3 | 3. Physics-II | 3 |
| 4. Expository Writing | 3 | 4. Probability and Probability Distribution-I | 3 |
| 5. Introduction of Zoology | 3(2+1) | 5. Probability and Probability Distribution-II | 3 |
| 6. Fundamental of Economics | 2 | 6. Software Packages | 3(2+1) |
| 7. Information and Communication Technologies (ICT) | 3(2+1) | 7. Classical Machenics | 3 |
| 8. Quantitative Reasoning-I | 3 |  |  |
| 9. Quantitative Reasoning-II | 3 |  |  |
| 10. Civic and Community Engagements | 2 |  |  |
| 11. Entrepreneurship | 2 |  |  |
| 12. Seerah and its Contemporary Applications | 2 |  |  |
| Total | 30 | Total | 21 |

# Scheme of Studies for the 2-Year Associate Degree/ BS (4-Year) in Mathematics 

(Semester-Wise Breakdown)
Semester-I

| Categories | Course Code | Course Title | Mark | Credit Hours |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gen Edu | ISL 112 / ETH 118 | Islamic Studies / Ethics (for Non- <br> Muslims) | 50 | 2 |  |  |  |  |  |
| Gen Edu | ENG 101 | Functional English | 100 | 3 |  |  |  |  |  |
| Gen Edu | ZOOL 101/PHYS 111 | Introductory Zoology/Basic Physics | 100 | $3(2+1)$ |  |  |  |  |  |
| Gen Edu | CS 110 | Introduction to Information and <br> Communication Technologies | 100 | $3(2+1)$ |  |  |  |  |  |
| Gen Edu | ECON 111/PSC 112 | Fundamental of Economics/Introduction <br> to Political Science | 50 | 2 |  |  |  |  |  |
| Major | MATH 121 | Calculus-I | 100 | 4 |  |  |  |  |  |
| Teaching of the Holy Quraan with Translation |  |  |  |  |  |  | Non-Credit |  |  |
| Total |  |  |  |  |  |  |  | $\mathbf{5 0 0}$ | $\mathbf{1 7}$ |

Semester-II

| Categories | Course Code | Course Title | Marks | Credit Hours |
| :---: | :---: | :---: | :---: | :---: |
| Gen Edu | ENG 102 | Introduction to Expository Writing | 100 | 3 |
| Gen Edu | QR 101 | Quantitative Reasoning-I (Mathematics) | 100 | 3 |
| Gen Edu | PSC 111 | Ideology and Constitution of Pakistan | 50 | 2 |
| Gen Edu | SOC 114 | Civic and Community Engagements | 50 | 2 |
| Gen Edu | ISC 113 |  <br> Seerah and its Contemporary Applications / any course form Arts \& Humanities Group (for Non-Muslims) | 50 | 2 |
| Major | MATH 122 | Calculus-II | 100 | 3 |
|  | Teaching of the Holy Quraan with Translation |  | Non-Credit |  |
| Total |  |  | 450 | 15 |

Semester-III

| Categories | Course Code | Course Title | Marks | Credit Hours |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gen Edu | MGT 215 | Entrepreneurship | 50 | 2 |  |  |  |  |
| Gen Edu | QR 102 | Quantitative Reasoning-II (Statistics) | 100 | 3 |  |  |  |  |
| Major | MATH 204 | Elements of Set Theory and <br> Mathematical Logic | 100 | 3 |  |  |  |  |
|  | MATH 205 | Discrete Mathematics | 100 | 3 |  |  |  |  |
| Major | Algebra-I | 100 | 3 |  |  |  |  |  |
| Major | MATH 223 | Calculus-III | 100 | 4 |  |  |  |  |
| Major | MATH 224 | Teaching of the Holy Quraan with Translation | Non-Credit |  |  |  |  |  |
| Total |  |  |  |  |  |  | $\mathbf{5 5 0}$ | $\mathbf{1 8}$ |

Semester-IV

| Categories | Course Code | Course Title | Marks | Credit Hours |
| :---: | :---: | :---: | :---: | :---: |
| Allied Course | CS 114 | Computer Programming | 100 | 3(2+1) |
| Major | MATH 226 | Algebra-II | 100 | 3 |
| Major | MATH 228 | Ordinary Differential Equations | 100 | 3 |
| Major | MATH 241 | Number Theory | 100 | 3 |
| Major | MATH 242 | Topology | 100 | 3 |
|  | Teaching of the Holy Quraan with Translation |  | Non-Credit |  |
| Total |  |  | 500 | 15 |

Semester-V

| Categories | Course Code | Course Title | Marks | Credit Hours |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Allied Course | PHYS 101 | Mechanics | 100 | 3 |  |  |  |  |  |
| Major | MATH 327 | Linear Algebra | 100 | 4 |  |  |  |  |  |
| Major | MATH 329 | Complex Analysis | 100 | 4 |  |  |  |  |  |
| Major | MATH 343 | Real Analysis-I | 100 | 3 |  |  |  |  |  |
| Major | MATH 344 | Mathematical Methods | 100 | 3 |  |  |  |  |  |
| Total |  |  |  |  |  |  | Non-Credit |  |  |
| Teaching of the Holy Quraan with Translation |  |  |  |  |  |  |  | $\mathbf{5 0 0}$ | $\mathbf{1 7}$ |

## Semester-VI

| Categories | Course Code | Course Title | Marks | Credit Hours |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Allied Course | STAT 215 | Probability and Probability <br> Distribution-I | 100 | 3 |  |  |  |  |
| Allied Course | PHYS 102 | Rotational Motional and <br> Thermodynamics | 100 | 3 |  |  |  |  |
| Allied Course | CS 406 | Software Packages | 100 | $3(2+1)$ |  |  |  |  |
| Major | MATH 325 | Affine Euclidean Geometry | 100 | 3 |  |  |  |  |
| Major | MATH 347 | Partial Differential Equations | 100 | 3 |  |  |  |  |
| Major | MATH 348 | Real Analysis-II | 100 | 3 |  |  |  |  |
| Teaching of the Holy Quraan with Translation |  |  |  |  |  | Non-Credit |  |  |
| Total |  |  |  |  |  |  | $\mathbf{6 0 0}$ | $\mathbf{1 8}$ |

## Semester-VII

| Categories | Course Code | Course Title | Marks | Credit Hours |
| :---: | :---: | :---: | :---: | :---: |
| Allied Course | STAT 345 | Probability and Probability <br> Distribution-II | 100 | 3 |
| Allied Course | PHYS 103 | Classical Mechanics-I | 100 | 3 |
| Major | MATH 449 | Numerical Analysis | 100 | 3 |
| Major | MATH 450 | Functional Analysis | 100 | 3 |
| Major | MATH | Elective-I | 100 | 3 |
| Major | MATH 499 | Field Experience/ Internship | 100 | 3 |


|  | Teaching of the Holy Quraan with Translation | Non-Credit |  |
| :---: | :---: | :---: | :---: |
| Total | $\mathbf{6 0 0}$ | $\mathbf{1 8}$ |  |

Semester-VIII

| Categories | Course Code |  | Course Title | Marks |
| :--- | :--- | :--- | :--- | :--- |
| Majer | Credit Hours |  |  |  |
| Major | MATH 440 | Integral Equation | 100 | 3 |
| Major | MATH 452 | Differential Geometry | 100 | 3 |
| Major | MATH- | Elective-II | 100 | 3 |
| Major | MATH- | Elective-III | 100 | 3 |
| Major | MATH 500 | Capstone Project/ Thesis | 100 | 3 |
| Teaching of the Holy Quraan with Translation |  |  |  | Non-Credit |
| Total |  |  |  | $\mathbf{5 0 0}$ |

## Note:

Courses included in the General Education Category are designed by the respective departments including their course codes, credit hours and titles (reflected in the scheme of studies). All such courses approved by the Syndicate are available on the university website (www.uom.edu.pk). For any query the office of the Registrar Academics may be approached for clarification/guidance.

## ELECTIVE COURSES

| S. No | Course Code | Course Name | Credit hours |
| :---: | :---: | :---: | :---: |
| 1 | MATH 461 | Mathematical Modeling | 03 |
| 2 | MATH 462 | Advanced Group Theory | 03 |
| 3 | MATH 463 | Optimization Theory | 03 |
| 4 | MATH 464 | Measure Theory | 03 |
| 5 | MATH 465 | Fluid Mechanics | 03 |
| 6 | MATH 466 | Stochastic Processes | 03 |
| 7 | PHYS 467 | Quantum Mechanics | 03 |
| 8 | MATH 468 | Special Relativity | 03 |
| 9 | MATH 469 | Heat and Mass Transfer | 03 |
| 10 | MATH 470 | Advanced Number Theory | 03 |
| 11 | MATH 471 | Analytical Dynamics | 03 |
| 12 | MATH 472 | Difference Equations | 03 |
| 13 | MATH 473 | Convex Analysis | 03 |
| 14 | MATH 474 | Econometrics | 03 |
| 15 | MATH 475 | Lie Algebra | 03 |
| 16 | MATH 476 | Galois Theory | 03 |
| 17 | MATH 477 | Simulations | 03 |
| 18 | MATH 478 | Dynamical System | 03 |
| 19 | MATH 479 | Mathematical Physics | 03 |
| 20 | MATH 480 | History of Mathematics | 03 |
| 21 | MATH 481 | Mathematical Biology | 03 |
| 22 | MATH 482 | Lie Group | 03 |
| 23 | MATH 483 | Rings and Modules | 03 |
| 24 | MATH 485 | Projective Geometry | 03 |

## COURSE CONTENTS

## SEMESTER-I

| Categories | Course Code | Course Title | Mark | Credit Hours |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gen Edu | ISL 112 / ETH 118 | Islamic Studies / Ethics (for Non- <br> Muslims) | 50 | 2 |  |  |  |  |
| Gen Edu | ENG 101 | Functional English | 100 | 3 |  |  |  |  |
| Gen Edu | ZOOL 101/PHYS 111 | Introductory Zoology/Basic Physics | 100 | $3(2+1)$ |  |  |  |  |
| Gen Edu | CS 110 | Introduction to Information and <br> Communication Technologies | 100 | $3(2+1)$ |  |  |  |  |
| Gen Edu | ECON 111/PSC 112 | Fundamental of Economics/Introduction <br> to Political Science | 50 | 2 |  |  |  |  |
| Major | MATH 121 | Calculus-I | 100 | 4 |  |  |  |  |
|  | Teaching of the Holy Quraan with Translation |  |  |  |  |  | $\mathbf{5 0 0}$ | $\mathbf{1 7}$ |

## MATH-121

## Calculus-I

Credit Hours: 4 (4+0)
Specific Objectives of course: Calculus serves as the foundation of advanced subjects in all areas of mathematics. This is the first course of Calculus. The objective of this course is to introduce students to the fundamental concepts of limit, continuity, differential and integral calculus of functions of one variable.

## Course Outline:

Equations and inequalities: Solving linear and quadratic equations, linear inequalities. Division of polynomials, synthetic division. Roots of a polynomial, rational roots; Viete Relations. Descartes rule of signs. Solutions of equations with absolute value sign. Solution of linear and non-linear inequalities with absolute value sign.
Functions and graphs: Domain and range of a function. Examples: polynomial, rational, piecewise defined functions, absolute value. Functions, and evaluation of such functions. Operations with functions: sum, product, quotient and composition. Graphs of functions: linear, quadratic, piecewise defined functions.

Lines and systems of equations: Equation of a straight line, slope and intercept of a line, parallel and perpendicular lines. Systems of linear equations, solution of system of linear equations. $\times$ Nonlinear systems: at least one quadratic equation.
Limits and continuity: Functions, limit of a function. Graphical approach. Properties of limits. Theorems of limits. Limits of polynomials, rational and transcendental functions. Limits at infinity, infinite limits, one-sided limits. Continuity.



Derivatives: Definition, techniques of differentiation. Derivatives of polynomials and rational, exponential, logarithmic and trigonometric functions. The chain rule. Implicit differentiation. Rates of change in natural and social sciences. Related rates. Linear approximations and differentials. Higher derivatives, Leibnitz's theorem.
Applications of derivatives: Increasing and decreasing functions. Relative extrema and optimization. First derivative test for relative extrema. Convexity and point of inflection. The second derivative test for extrema. Curve sketching. Mean value theorems. Indeterminate forms and L'Hopitals rule. Inverse functions and their derivatives.
Integration: Anti derivatives and integrals. Riemann sums and the definite integral. Properties of Integral. The fundamental theorem of calculus. The substitution rule.

## Recommended Books:

1. Thomas, Calculus, 11th Edition. Addison Wesley Publishing Company, 2005
2. H. Anton, I. Bevens, S. Davis, Calculus, 8th Edition, John Wiley \& Sons, Inc. 2005
3. Hughes-Hallett, Gleason, McCallum, et al, Calculus Single and Multivariable, 3rd Edition. John Wiley \& Sons, Inc. 2002.
4. Frank A. Jr, Elliott Mendelson, Calculus, Schaum"cs outlines series, 4th Edition, 1999
5. C.H. Edward and E.D Penney, Calculus and Analytics Geometry, Prentice Hall, Inc. 1988
6. E. W. Swokowski, Calculus with Analytic Geometry, PWS Publishers, Boston, Massachusetts, 1983.
7. M. Liebeck, A Concise introduction to pure Mathematics, CRC Press, 2011.
8. A. Kaseberg, Intermediate Algebra, Thomson Brooks/cole, 2004.



## Semester-II

| Categories | Course Code | Course Title | Marks | Credit Hours |
| :---: | :---: | :---: | :---: | :---: |
| Gen Edu | ENG 102 | Introduction to Expository Writing | 100 | 3 |
| Gen Edu | QR 101 | Quantitative Reasoning-I (Mathematics) | 100 | 3 |
| Gen Edu | PSC 111 | Ideology and Constitution of Pakistan | 50 | 2 |
| Gen Edu | SOC 114 | Civic and Community Engagements | 50 | 2 |
| Gen Edu | ISC 113 | سيرت رسول (1) اور اس كى عصرى مغويت <br> Seerah and its Contemporary Applications / any course form Arts \& Humanities Group (for Non-Muslims) | 50 | 2 |
| Major | MATH 122 | Calculus-II | 100 | 3 |
|  | Teaching of the Holy Quraan with Translation |  | Non-Credit |  |
| Total |  |  | 450 | 15 |

## QR-101 Quantitative Reasoning-I

Credit Hours: 3

## COURSE INTRODUCTION

This course is based on quantitative reasoning 1 course. It will enhance the quantitative reasoning skills learned in quantitative reasoning 1 course. Students will be introduced to more tools necessary for quantitative reasoning skills to live in the fast paced 21st century. Students will be introduced to importance of mathematical skills in different professional settings, social and natural sciences. These quantitative reasoning skills will help students to better participate in national and international issues like political and health issues. This course will prepare the students to apply quantitative reasoning tools more efficiently in their professional and daily life activities. This course will help them to better understand the information in form of numeric, graphs, tables, and functions.

## COURSE OBJECTIVES

- Students will be introduced to the above listed concepts, and they will be prepared to apply these concepts to practical life scenarios.
- This course will enhance their ability to deal with scenarios involving quantitative reasoning skills in a logical manner which they can face in their practical lives
- It will prepare students to deal with different forms of data occurring in professional, social and natural sciences
- Students will be introduced to scenarios involving functions and probability in different disciplines.
- This course will prepare the students to apply the quantitative reasoning skills in other disciplines.
- This course will provide solid foundation for students to use the quantitative reasoning skills in solving practical life problems.


## COURSE MODULE:

1. Investigating relationships between variables.
2. Exploring tools to find relationship between variables, Resources, and population growth: dealing with economic, environmental, and social issues
3. Graphical and analytical approaches to solve a problem.

4. Applications of graphical \& analytical approaches to solve social \& economic problems.
5. Understanding inequalities around us
6. Dealing with practical problems involving inequalities in different disciplines
7. Golden ratio in sculptures
8. Comparison of statements and their use in social and economic problems,
9. Sequence
10. Survival in the modern World,
11. Propositions and truth values,
12. Categorical proposition, and its applications
13. Methods to explore and summarize data, drawing graphs and identifying misleading graphs,
14. Methods to discuss the basic characteristics of any datasets, like finding a most representative value in a data, and methods to measure the amount of spread of a data,
15. Methods to measure degree of relationship among variables, finally this module includes methods to Count the odds.

## RECOMMENDED TEXTBOOKS:

1. Bennett, J. \& Briggs, W. (2015). Using and understanding mathematics (6th Edition). Pearson Education, Limited.
2. Blitzer, R. (2014). Pre-calculus. (5th Edition). Pearson Education, Limited
3. Mathematical thinking and reasoning 2008 by Aufmann, Lockwood, Nation \& Clegg published by Houghton Mifflin Company USA.
4. Precalculus Graphical, Numerical, Algebraic 8th edition by Franklin D. Demana, Bert K. Waits, Gregory D. Foley \& Daniel Kennedy published by Addison Wesley USA.
5. Precalculus Mathematics for Calculus, 6th edition by James Stewart, Lothar Redlin and Saleem Watson published by Brooks/Cole Cengage Learning USA.
6. GRE Math Review https://www.ets.org/s/gre/pdf/gre_math_review.pdf
7. OpenAlgebra.com
8. A free math study guide with notes and YouTube video tutorials.

Source: The course manual and teacherces guide are available at the HEC website.

## MATH-122 Calculus-II

Credit Hours: 3+0
Specific Objectives of course: This is second course of Calculus. As continuation of Calculus I, it focuses on techniques of integration and applications of integrals. The course also aims at introducing the students to infinite series, parametric curves and polar coordinates.

## Course Outline:

Techniques of integration: Integrals of elementary, hyperbolic, trigonometric, logarithmic and exponential functions. Integration by parts, substitution and partial fractions. Approximate integration. Improper integrals. Gamma functions.
Applications of integrals: Area between curves, average value. Volumes. Arc length. Area of a surface of revolution. Applications to Economics, Physics, Engineering and Biology.
Infinite series: Sequences and series. Convergence and absolute convergence.
Tests for convergence: divergence test, integral test, p- series test, comparison test, limit comparison test, alternating series test, ratio test, root test. Power series. Convergence of power series. Representation of functions as power series. Differentiation and integration of power series. Taylor and McLaurin series. Approximations by Taylor polynomials.

Conic section, parameterized curves and polar coordinates: Curves defined by parametric equations.
Calculus with parametric curves: tangents, areas, arc length. Polar coordinates. Polar curves, tangents to polar curves. Areas and arc length in polar coordinates.

## Recommended Books:

1. G.B. Thomas, Calculus, 11th Edition. Addison Wesley Publishing Company, 2005
2. H. Anton, I. Bevens, S. Davis, Calculus, 8th Edition, John Wiley \& Sons, Inc. 2005
3. Hughes-Hallett, Gleason, McCallum, et al, Calculus Single and Multivariable, 3 rd Edition. John Wiley \& Sons, Inc. 2002.
4. Frank A. Jr, Elliott Mendelson, Calculus, Schaum"cs outlines series, 4th Edition, 1999
5. C.H. Edward and E.D Penney, Calculus and Analytics Geometry, Prentice Hall, Inc. 1988
6. E. W. Swokowski, Calculus with Analytic Geometry, PWS Publishers, Boston, Massachusetts, 1983.
7. M. Liebeck, A Concise introduction to pure Mathematics, CRC Press, 2011.
8. A. Kaseberg, Intermediate Algebra, Thomson Brooks/COLE, 2004.J. Stewart, Calculus ( $5^{\text {th }}$ edition or latest edition), Brooks/ Cole, 2002.
9. J. Stewart, Calculus early transcendentals, 7th Edition, Brooks/COLE, 2008.


CHAIRMAN

Semester-III

| Categories | Course Code | Course Title | Marks | Credit Hours |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gen Edu | MGT 215 | Entrepreneurship | 50 | 2 |  |  |  |  |  |
| Gen Edu | QR 102 | Quantitative Reasoning-II (Statistics) | 100 | 3 |  |  |  |  |  |
| Major | MATH 204 | Elements of Set Theory and <br> Mathematical Logic | 100 | 3 |  |  |  |  |  |
| Major | MATH 205 | Discrete Mathematics | 100 | 3 |  |  |  |  |  |
| Major | MATH 223 | Algebra-I | 100 | 3 |  |  |  |  |  |
| Major | MATH 224 | Calculus-III | 100 | 4 |  |  |  |  |  |
| Teaching of the Holy Quraan with Translation |  |  |  |  |  |  | Non-Credit |  |  |
| Total |  |  |  |  |  |  |  | $\mathbf{5 5 0}$ | $\mathbf{1 8}$ |

## MATH-204

## Elements of Set Theory and Mathematical Logic

Credit Hours: 3+0
Specific Objectives of course: Everything mathematicians do can be reduced to statements about sets, equality and membership which are basics of set theory. This course introduces these basic concepts. The course aims at familiarizing the students with cardinals, relations and fundamentals of propositional and predicate logics.
Course Outline: Set theory: Sets, subsets, operations with sets: union, intersection, difference, symmetric difference, Cartesian product and disjoint union.
Functions: Graph of a function. Composition; injections, surjections, bijections, inverse function.
Computing cardinals: Cardinality of Cartesian product, union. Cardinality of all functions from a set to another set. Cardinality of all injective, surjective and bijective functions from a set to another set. Infinite sets, finite sets. Countable sets, properties, examples $(Z, Q) . R$ is not countable. $R, R \times R, R \times R \times R$ have the same cardinal. Operations with cardinal numbers. CantorBernstein theorem.
Relations: Equivalence relations, partitions, quotient set; examples, parallelism, similarity of triangles. Order relations, min, max, inf, sup; linear order. Examples: $N, Z, R, P(A)$. Well ordered sets and induction. Inductively ordered sets and Zorn"s lemma.
Mathematical logic: Propositional Calculus. Truth tables. Predicate Calculus.

## Recommended Books:

1. M. Liebeck, A Concise Introduction to Pure Mathematics, CRC Press, 2011.
2. N. L. Biggs, Discrete Mathematics, Oxford University Press, 2002.
3. R. Garnier, J. Taylor, Discrete Mathematics, Chapters $1,3,4,5$, CRC Press, 2010.
4. A.A. Fraenkal, Abstract Set Theory, North-Holland Publishing Company, 1966.
5. P. Suppes, Axiomatic Set Theory, Dover Publication, 1972.
6. P.R. Halmos, Naive Set Theory, New York, Van Nostrand, 1950.
7. B. Rotman, G.T. Kneebone, The Theory of sets and Transfinite Numbers, Oldbourne London, 1968.
8. D. Smith, M. Eggen, R.St. Andre, A Transition to Advanced Mathematics, Brooks/Cole, 2001.


## MATH-224 Calculus-III

Credit Hours: 4+0

## Specific Objectives of the Course:

Multivariate calculus is serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes the basic concepts and skills needed for mathematical manipulation. The main focus will be on the study of functions having two, three or more variables.
Course Outline: Vectors and analytic geometry in space: Coordinate system. Rectangular, cylindrical and spherical coordinates. The dot product, the cross product. Equations of lines and planes. Quadric surfaces.
Vector-valued functions: Vector-valued functions and space curves. Derivatives and integrals of vector valued functions. Arc length. Curvature, normal and binormal vectors.
Multivariable functions and partial derivatives: Functions of several variables. Limits and Continuity. Partial derivatives, Composition and chain rule. Directional derivatives and the gradient vector. Implicit function theorem for several variables. Maximum and minimum values. Optimization problems. Lagrange Multipliers.
Multiple integrals: Double integrals over rectangular domains and iterated integrals. Nonrectangular domains. Double integrals in polar coordinates. Triple integrals in rectangular, cylindrical and spherical coordinates. Applications of double and triple integrals. Change of variables in multiple integrals. Vector calculus: Vector fields. Line integrals. Green's theorem. Curl and divergence. Surface integrals over scalar and vector fields. Divergence theorem. Stokes' theorem.

## Recommended Books:

1. Thomas, Calculus, 11th Edition. Addison Wesley Publishing Company, 2005
2. . H. Anton, I. Bevens, S. Davis, Calculus, 8th Edition, John Wiley \& Sons, Inc. 2005
3. Hughes-Hallett, Gleason, McCallum, et al, Calculus Single and Multivariable, 3 rd Edition. John Wiley \& Sons, Inc. 2002.
4. Frank A. Jr, Elliott Mendelson, Calculus, Schaum"s outlines series, 4 th Edition, 1999
5. C.H. Edward and E.D Penney, Calculus and Analytics Geometry, Prentice Hall, Inc. 1988
6. E. W. Swokowski, Calculus with Analytic Geometry, PWS Publishers, Boston, Massachusetts, 1983.
7. M. Liebeck, A Concise introduction to pure Mathematics, CRC Press, 2011.
8. A. Kaseberg, Intermediate Algebra, Thomson Brooks/COLE, 2004.
9. J. Stewart, Calculus early transcendentals, 7th Edition, Brooks/COLE, 2008.

## MATH-205 Discrete Mathematics

Credit Hours: 3+0
Specific Objectives of the Course:
This course shall assume background in number theory. It lays a strong emphasis on understanding and utilizing various strategies for composing mathematical proof.



## Course outline:

Counting methods: Basic methods: product, inclusion-exclusion formulae. Permutations and combinations. Recurrence relations and their solutions. Generating functions. Double counting. Applications. Pigeonhole principle, applications.
Relations: Binary relations, n-ary Relations. Closures of relations. Composition of relations, inverse relation.
Graphs: Graph terminology. Representation of graphs. Graphs isomorphism.
Algebraic methods: the incidence matrix. Connectivity, Eulerian and Hamiltonian paths.
Shortest path problem. Trees and spanning trees. Complete graphs and bivalent graphs.

## Recommended Books:

1. B. Bollobas, Graph Theory, Springer Verlag, New York, 1979.
2. K.R. Parthasarathy, Basic Graph Theory, McGraw-Hill, 1994
3. K.H. Rosen, Discrete Mathematics and its Application, McGraw-Hill, 6th edition, 2007.
4. B. Kolman, R.C. Busby, S.C. Ross, Discrete Mathematical Structures, Prentice-Hall of India, New Delhi, 5th edition, 2008.
5. A. Tucker, Applied Combinatorics, John Wiley and Sons, Inc New York, 2002.
6. R. Diestel, Graph Theory, 4th edition, Springer- Verlag, New York, 2010.
7. N.L. Brigs, Discrete Mathematics, Oxford University Press, 2003
8. K.A. Ross, C.R.B. Wright, Discrete Mathematics, Prentice Hall, New Jersey, 2003.

## MATH-223 Algebra-I (Group Theory)

Credit Hours: 3
Specific Objectives of course: This course introduces basic concepts of groups and their homomorphisms. The main objective of this course is to prepare students for courses which require a good background in group theory like Rings and Modules, Linear Algebra, Group Representation, Galois Theory etc.

## Course Outline:

Groups: Definition of a group, subgroup, subgroup generated by a set. The cyclic groups, cosets and Lagrange"s theorem. Normalizer centralizer. The center of a group. Equivalence relation in a group, conjugacy classes. Normal subgroups, quotient group.
Group homomorphisms: Homomorphisms and isomorphism and Automorphism. Kernel and image of homomorphism. Isomorphism theorems. Permutation groups. The cyclic decomposition of a permutation group. Cayley"s theorem. Direct product of two groups and examples.

## Recommended Books:

1. J. Rose, A Course on Group Theory, Cambridge University Press, 1978.
2. I. N. Herstein, Topics in Algebra, Xerox Publishing Company, 1964.
3. P. M. Cohn, Algebra, John Wiley and Sons, London, 1974.
4. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Cambridge University Press, 1986.
5. J. B. Fraleigh, A First Course in Abstract Algebra, Addison- Wesley Publishing Company, 2002.


6. . Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
7. D. S. Dummit and R. M. Foote, Abstract Algebra, 3rd Edition, Addison-Wesley Publishing Company, 2004.
8. A. Majeed, Group Theory, Ilmi kitab Khana, 2005.
9. J. B. Farleigh, A First Course in Abstract Algebra (7 ${ }^{\text {th }}$ edition), Addison-Wesley, 2000.
10. K. H. Dar, Abstract Algebra, Ilmi Kitab Khana Lahore, 2000.


## Semester-IV

| Categories | Course Code | Course Title | Marks | Credit Hours |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Allied <br> Course | CS 114 | Computer Programming | 100 | $3(2+1)$ |  |  |  |  |
| Major | MATH 226 | Algebra-II | 100 | 3 |  |  |  |  |
| Major | MATH 228 | Ordinary Differential Equations | 100 | 3 |  |  |  |  |
| Major | MATH 241 | Number Theory | 100 | 3 |  |  |  |  |
| Major | MATH 242 | Topology | 100 | 3 |  |  |  |  |
| Teaching of the Holy Quraan with Translation |  |  |  |  |  |  | Non-Credit |  |
| Total |  |  | $\mathbf{5 0 0}$ | $\mathbf{1 5}$ |  |  |  |  |

## MATH-226 Algebra-II (Ring Theory and Fields)

## Specific Objectives of the Course:

Specific Objectives of course: This is a course in advanced abstract algebra, which builds on the concepts learnt in Algebra I. The objectives of the course are to introduce students to the basic ideas and methods of modern algebra and enable them to understand the idea of a ring and an integral domain and be aware of examples of these structures in mathematics; appreciate and be able to prove the basic results of ring theory; appreciate the significance of unique factorization in rings and integral domains.

## Course Outline:

Rings: Definition, examples. Quadratic integer rings. Examples of non-commutative rings. The Hamilton quaternions. Polynomial rings. Matrix rings. Units, zero-divisors, nilpotents, idempotents. Subrings, Ideals. Maximal and prime Ideals.
Left, right and two-sided ideals: Operations with ideals. The ideal generated by a set. Quotient rings. Ring homomorphism. The isomorphism theorems, applications. Finitely generated ideals. Rings of fractions.
Integral Domain: The Chinese remainder theorem. Divisibility in integral domains, greatest common divisor, least common multiple. Euclidean domains. The Euclidean algorithm. Principal ideal domains. Prime and irreducible elements in an integral domain. Gauss lemma, irreducibility criteria for polynomials. Unique factorization domains. Finite fields. Polynomials in several variables. Symmetric polynomials. The fundamental theorem of symmetric polynomials.

## Recommended Books:

1. J. Rose, A Course on Group Theory, Cambridge University Press, 1978.
2. I. N. Herstein, Topics in Algebra, Xerox Publishing Company, 1964.
3. . P. M. Cohn, Algebra, John Wiley and Sons, London, 1974
4. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Cambridge University Press, 1986.
5. J. B. Fraleigh, A First Course in Abstract Algebra, Addison- Wesley Publishing Company, 2002.
6. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
7. D. S. Dummit and R. M. Foote, Abstract Algebra, 3rd Edition, Addison-Wesley Publishing Company, 2004.
8. S.Lipschutz, Schaum's Outline of Linear Algebra (last Edition), 2000.

## MATH-228 Ordinary Differential Equations

Specific Objectives of course: To introduce students to the formulation, classification of differential equations and existence and uniqueness of solutions. To provide skill in solving initial value and boundary value problems. To develop understanding and skill in solving first and second order linear homogeneous and non- homogeneous differential equations and solving differential equations using power series methods.
Course Outline: Preliminaries: Introduction and formulation, classification of differential equations, existence and uniqueness of solutions, introduction of initial value and boundary value problems
First order ordinary differential equations: Basic concepts, formation and solution of differential equations. Separable variables, Exact Equations, Homogeneous Equations, Linear equations, integrating factors. Some nonlinear first order equations with known solution, differential equations of Bernoulli and Ricaati type, Clairaut equation, modeling with first-order ODEs, Basic theory of systems of first order linear equations, Homogeneous linear system with constant coefficients, Non homogeneous linear system
Second and higher order linear differential equations: Initial value and boundary value problems, Homogeneous and non-homogeneous equations, Superposition principle, homogeneous equations with constant coefficients, Linear independence and Wronskian, Nonhomogeneous equations, undetermined coefficients method, variation of parameters, CauchyEuler equation, Modeling.
Sturm-Liouville problems: Introduction to eigen value problem, adjoint and self adjoint operators, self adjoint differential equations, eigen values and eigen functions, Sturm-Liouville (S-L) boundary value problems, regular and singular S-L problems, properties of regular S-L problems
Series Solutions: Power series, ordinary and singular points, Existence of power series solutions, power series solutions, types of singular points, Frobenius theorem, Existence of Frobenius series solutions, solutions about singular points, The Bessel, modified Bessel Legendre and Hermite equations and their solutions.

## Recommended Books:

1. Dennis G. Zill and Michael R., Differential equations with boundary-value problems by Cullin 5th Edition Brooks/Cole, 1997.
2. William E. Boyce and Richard C. Diprima, Elementary differential equations and boundary value problems, Seventh Edition John Wiley \& Sons, Inc
3. V. I. Arnold, Ordinary Differential Equations, Springer, 1991.
4. T. Apostol, Multi Variable Calculus and Linear Algebra, 2nd ed., John Wiley and sons, 1997.

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## MATH-241 Number Theory

Specific Objectives of course: The focus of the course is on study of the fundamental properties of integers and develops ability to prove basic theorems. The specific objectives include study of division
algorithm, prime numbers and their distributions, Diophantine equations, and the theory of congruences.
Course Outline:
Preliminaries: Well-ordering principle. Principle of finite induction.
Divisibility theory: The division algorithms. Basis representation theorem. Prime and composite numbers. Canonical decomposition. The greatest common divisor. The Euclidean algorithm. The fundamental theorem of arithmetic. Least common multiple.
Linear Diophantine equations: Congruences. Linear congruences. System of linear congruences. The Chinese remainder theorem. Divisibility tests. Solving polynomial congruences. Fermat's and Euler's theorems. Wilson's theorem.
Arithmetic functions: Euler's phi-function. The functions of J and sigma. The Mobius function. The sieve of Eratosthenes. Perfect numbers. Fermat and Mersenne primes.
Primitive Roots and Indices: The order of an integer mod $n$, Primitive roots for primes. Composite numbers having primitive roots.
Quadratic residues: Legendre symbols and its properties. The quadratic reciprocity law. Quadratic congruences with composite moduli. Pythagorean triples. Representing numbers as sum of two squares.

## Recommended Books:

1. D.M. Burton, Elementary Number Theory, McGraw-Hill, 2007.
2. W.J. Leveque, Topics in Number Theory, vols. I and II, Addison- Wesley, 1956.
3. S.B. Malik, Basic Number Theory, Vikas Publishing house, 1995.
4. K.H. Rosen, Elementary Number Theory and its Applications, 5th edition, AddisonWesley, 2005.
5. I. Niven, H.S. Zuckerman, H.L. Montgomery, An Introduction to the theory of Numbers, John Wiley and Sons, 1991.
6. A. Adler, J.E. Coury, The Theory of Numbers, Jones and Bartlett Publishers, 1995.

## MATH-242 Topology

## Specific Objectives of the Course:

The aim of this course is to introduce the students to metric spaces and topological spaces. After completion of this course, they would be familiar with separation axioms, compactness and completeness. They would be able to determine whether a function defined on a metric or topological space is continuous or not and what homeomorphisms are.

## Course Outline:

Topological spaces: Introduction to topological and metric spaces, open closed derived sets, neighborhoods. Examples. Limit points and accumulation points. Interior, closure, dense subsets bases and sub base for topological and matric spaces. .


Constructing new topological spaces: Cartesian products, induced topology and quotient topology. Relative topology, Continuous maps, open and closed maps, homeomorphisms. Examples: R, RxR, $S^{\wedge} 1, S^{\wedge} 2$, torus, cylinder, Cauchy sequences, complete metric spaces. Separation axioms, Compact spaces. Compactness and completeness of metric spaces, connected components. Properties. Image of a connected set through a continuous map. Pathconnectedness. Connected spaces, connected components. Properties. Image of a connected set through a continuous map. Path-connectedness.

## Recommended Books:

1. J. Kelly, General Topology, Springer, 2005.
2. . K. Janich, Topology, Springer, 1994.
3. J. Hocking, G. Young, Topology, Dover Publications, 1961.
4. J. R. Munkres, Topology - A First Course, Prentice-Hall, 2003.
5. . G. Simmons, Topology and modern analysis, McGraw-Hill, 1963.
6. S. Lipschutz, General Topology, McGraw-Hill, 2004.
7. .J. Dugundji, Topology, Allyn and Bacon, 1966.


Semester-V

| Categories | Course Code | Course Title | Marks | Credit Hours |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Allied Course | PHYS 101 | Mechanics | 100 | 3 |  |  |  |
| Major | MATH 327 | Linear Algebra | 100 | 4 |  |  |  |
| Major | MATH 329 | Complex Analysis | 100 | 4 |  |  |  |
| Major | MATH 343 | Real Analysis-I | 100 | 3 |  |  |  |
| Major | MATH 344 | Mathematical Methods | 100 | 3 |  |  |  |
| Teaching of the Holy Quraan with Translation |  |  |  |  |  | Non-Credit |  |
| Total |  | $\mathbf{5 0 0}$ | $\mathbf{1 7}$ |  |  |  |  |

## PHYS-101 Mechanics

## Course Objectives:

This course is based on basic concepts of physics that will enable students to understand fundamentals of mechanics. After studying this course students will be able to solve problems related to physics/mechanics.

## Course Outline:

Motion along a straight line (position, displacement, velocity, acceleration and graphical analysis of motion). Vectors and their components (vectors, unit vectors, vectors addition by components, multiplication of vectors). Motion in two and three dimensions (position, velocity etc. projectile motion, uniform circular motion, relative motion in one and two dimensions). Force and Motion-I (Newton"s laws, applying Newton"s laws, some particular forces). Force and Motion-II (Friction, Drag force and terminal speed, Uniform circular motion). Kinetic Energy and Work (work done and Kinetic energy, work done by gravitational force, Work done by a spring and variable force, Power). Potential Energy and Conservation of Energy (Potential energy, conservation of mechanical energy, potential energy curve, Work done on a system by external force). Center of mass and linear momentum(center of mass, Newton"s second law for a system of particles, linear momentum, Collision and impulse, Conservation of linear momentum, Momentum and kinetic energy in collision, elastic collision in one and two dimensions, systems with varying mass

Text Book: Fundamentals of Physics by Halliday, Resnick and Walker: $8^{\text {th }}$ Edition (Text book).
Holiday, Resnik, Krane
Serway Jewett, Physics for Scientists and Engineers with Modern Physics. (2010)
Recommended Book:
University Physics with Modern Physics $13^{\text {th }}$ Ed. Pearson Education Inc. USA


## MATH -327 Algebra-III (Linear Algebra)

## Specific Objectives of the Course:

Linear algebra is the study of vector spaces and linear transformations. The main objective of this course is to help students learn in rigorous manner, the tools and methods essential for studying the solution spaces of problems in mathematics, engineering, the natural sciences, and social sciences and develop mathematical skills needed to apply these to the problems arising within their field of study; and to various real world problems.

## Course Outline:

System of Linear Equations: Representation in matrix form. Matrices. Operations on matrices. Echelon and reduced echelon form. Inverse of a matrix (by elementary row operations). Solution of linear system. Gauss-Jordan method. Gaussian elimination.
Determinants: Permutations of order two and three and definitions of determinants of the same order. Computing of determinants. Definition of higher order determinants. Properties.
Vector Spaces: Definition and examples, subspaces. Linear combination and spanning set. Linearly Independent sets. Finitely generated vector spaces. Bases and dimension of a vector space. Operations on subspaces, Intersections, sums and direct sums of subspaces. Quotient Spaces.
Linear mappings: Definition and examples. Kernel and image of a linear mapping. Rank and nullity. Reflections, projections. Change of basis. Eigen-values and eigenvectors. Theorem of Hamilton-Cayley.
Inner product Spaces: Definition and examples. Properties, Projection. Cauchy inequality. Orthogonal and orthonormal basis. Gram Schmidt Process. Diagonalization.

## Recommended Books:

1. Ch. W. Curtis, Linear Algebra, Springer 2004.
2. T. Apostol, Multi Variable Calculus and Linear Algebra, 2nd ed., John Wiley and sons, 1997.
3. H. Anton, C. Rorres, Elementary Linear Algebra: Applications Version, 10th Edition, John Wiley and sons, 2010.
4. S. Friedberg, A. Insel, Linear Algebra, 4th Edition, Pearson Education Canada, 2003. S. I. Grossman, Elementary Linear Algebra, 5th Edition, Cengage Learning, 2004.

## MATH-329 Complex AnalysisSpecific

## Objectives of the Course:

This is an introductory course in complex analysis, giving the basics of the theory along with applications, with an emphasis on applications of complex analysis and especially conformal mappings. Students should have a background in real analysis (as in the course Real Analysis I), including the ability to write a simple proof in an analysis context.

## Course Outline:

Introduction: The algebra of complex numbers, Geometric representation of complex numbers,



Powers and roots of complex numbers.
Functions of Complex Variables: Definition, limit and continuity, Branches of functions, Differentiable and analytic functions. The Cauchy-Riemann equations, Entire functions, Harmonic functions
Elementary functions: The exponential, Trigonometric, Hyperbolic, Logarithmic and Inverse elementary functions, Open mapping theorem. Maximum modulus theorem.
Complex Integrals: Contours and contour integrals, Cauchy-Goursat theorem, Cauchy integral formula, Lioville"s theorem, Morerea"s theorem.
Series: Power series, Radius of convergence and analyticity, Taylor"s and Laurent"s series, Integration and differentiation of power series. Singularities,
Poles and residues: Zero, singularities, Poles and Residues, Types of singular points, Calculus of residues, contour integration, Cauchy"s residue theorem with applications. Mobius transforms, Conformal mappings and transformations.

## Recommended Books:

1. R. V. Churchill, J. W. Brown, Complex Variables and Applications, 5 th edition, McGraw Hill, New York, 1989.
2. J. H. Mathews and R. W. Howell, Complex Analysis for Mathematics and Engineering, 2006.
3. S. Lang, Complex Analysis, Springer-Verlag, 1999.
4. R. Remmert, Theory of Complex Functions, Springer-Verlag, 1991.
5. W. Rudin, Real and Complex Analysis, McGraw-Hill, 1987.
S. Murray, S. Lipschutz, D. Spellman, Complex Analysis, Schaum"s Outlines Series, 2000.

## MATH-343 Real Analysis-I

## Specific Objectives of the Course:

This is the first course in analysis. It develops the fundamental ideas of analysis and is aimed at developing the students" ability in reading and writing mathematical proofs. Another objective is to provide sound understanding of the axiomatic foundations of the real number system, in particular the notions of completeness and compactness.

## Course Outline:

Number Systems: Ordered fields. Rational, real and complex numbers. Archimedean property, supremum, infimum and completeness. Topology of real numbers: Convergence, completeness, completion of real numbers. Open sets, closed sets, compact sets. Heine Borel Theorem. Connected sets.
Sequences and Series of Real Numbers: Limits of sequences, algebra of limits. Bolzano Weierstrass Theorem. Cauchy sequences, liminf, limsup. Limits of series, convergences tests, absolute and conditional convergence. Power series.
Continuity: Functions, continuity and compactness, existence of minimizers and maximizers, uniform continuity. Continuity and connectedness, Intermediate mean Value Theorem. Monotone functions and discontinuities.
Differentiation: Mean Value Theorems, L"Hopital"s Rule, Taylor"s Theorem.



## Recommended Books:

1. S. Lang, Analysis I, Addison-Wesley Publ. Co., Reading, Massachusetts, 1968.
2. W. Rudin, Principles of Mathematical Analysis, 3rd ed., Mc.Graw-Hill, 1976
3. B. S. Thomson, J. B. Bruckner and A. M. Bruckner, Elementary Real Analysis, 2nd Ed. 2008.
4. G. Boros, V. Moll, Irresistible Integrals: Symbolics, Analysis an Experiments in the Evaluation of Integrals, Cambridge University Press, 2004.
5. J. Borwein, D. Bailey, R. Girgenson, Experimentation in Mathematics: Computational Paths to discovery, Wellesley, MA, A.K. Peters, 2004.
6. G. Bartle, R. Sherbert , Introduction to Real Analysis, 3 rd edition, John Wiley, New York, 1999.
7. S. C. Malik, Mathematical Analysis, New Delhi University, 2010.

## MATH-344 Mathematical Methods

## Specific Objectives of the Course:

The purpose of this course is to teach students, various methods and techniques for solving differential equations of applied nature and applications.
Course Outline:
Fourier Methods: The Fourier transforms. Fourier analysis of the generalized functions. The Laplace transforms. Hankel transforms for the solution of PDEs and their application to boundary value problems.
Green's Functions and Transform Methods: Expansion for Green"s functions. Transform methods. Closed form Green"s functions.
Perturbation Techniques: Perturbation methods for algebraic equations. Perturbation methods for differential equations.
Variational Methods: Euler-Lagrange equations. Integrand involving one, two, three and $n$ variables. Special cases of Euler-Lagrange"s equations. Necessary conditions for existence of an extremum of a functional. Constrained maxima and minima.

## Recommended Books:

1. D. L. Powers, Boundary Value Problems and Partial Differential Equations, 5th edition, Academic Press, 2005.
2. W. E. Boyce, Elementary Differential Equations, 8th edition, John Wiley and Sons, 2005.
3. M. L. Krasnov, G. I. Makarenko and A. I. Kiselev, Problems and Exercises in the Calculus of Variations, Imported Publications, Inc., 1985.
4. J. W. Brown and R. V. Churchil, Fourier Series and Boundary Value Problems, McGraw Hill, 2006.
5. A. D. Snider, Partial Differential Equations: Sources and Solutions, Prentice Hall Inc., 1999.



## Semester-VI

| Categories | Course Code | Course Title | Marks | Credit Hours |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Allied Course | STAT 215 | Probability and Probability <br> Distribution-I | 100 | 3 |  |  |  |
| Allied Course | PHYS 102 | Rotational Motional and <br> Thermodynamics | 100 | 3 |  |  |  |
| Allied Course | CS 406 | Software Packages | 100 | $3(2+1)$ |  |  |  |
| Major | MATH 325 | Affine Euclidean Geometry | 100 | 3 |  |  |  |
| Major | MATH 347 | Partial Differential Equations | 100 | 3 |  |  |  |
| Major | MATH 348 | Real Analysis-II | 100 | 3 |  |  |  |
| Teaching of the Holy Quraan with Translation |  |  |  |  |  | Non-Credit |  |
| Total |  | $\mathbf{6 0 0}$ | $\mathbf{1 8}$ |  |  |  |  |

## STAT-317 Probability and Probability Distributions-I

## Course Objectives:

- This course is designed to give students a conceptual knowledge of discrete random variables and probability theory.
- This course provides the fundamentals of probability theory in different disciplines.
- This course helps to model the uncertain behavior from the real life scenario.


## Learning Outcomes:

- Understand the basic concepts and applications of probability.
- Investigate the nature of stochastic process and apply suitable probability distributions for the random variable generated from such process.
- Find probabilities using probability distributions.
- Use probability concepts and laws in decision analysis.


## Course Contents:

Basic Functions and Inequalities: Distribution function, Probability mass and density functions. Location, scale, and shape parameters. Joint and conditional distributions for two and more random variables, Marginal and conditional distributions, Mathematical expectation and its properties, Conditional Expectation, Moments, Stochastic independence, Moment Generating Function, Comulant Generating function of distributions Characteristic function, Factorial Moments, Cummulants, L moments and their relationships. Probability Generating Function,

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Relation between Moments and Cumulants.
Discrete Probability distributions: Discrete Uniform, Bernoulli, Binomial, Multinomial, Negative Binomial, Geometric, Hypergeometric, Poisson, Discrete Weibull distributions with properties (mean, variance, moments, generating functions)

## Books Recommended

i. Forbes, C., Evans M., Hastings, N., Peacock, B. (2010) "Statistical Distributions" $4^{\text {th }}$ Edition, Jhon Wiely and Sons Inc.. Hobokon, New Jersey.
ii. Hirai, A.S. (1998), "A Course in Mathematical Statistics", Ilmi Kutab Khana, Lahore.
iii. Mood, A.M, Graybill, F.A. and Boss, D.C. (1997), "Introduction to the Theory of Statistics", McGraw Hill, New York.
iv. Khan, M. K., (1996). "Probability with Applications", Maktiba Ilmi, Lahore.
v. Hogg, R.M. and Craig, A.T. (1995), "Introduction to Mathematical Statistics". Prentice Hall, Engle wood Cliffs, New Jersey.

## PHYS-102 Rotational Motion and Thermodynamics

## Cr.hr: 03

## Course Objectives:

This course is based on basic concepts of physics that will enable students to understand fundamentals of rotational mechanics and thermodynamics. After studying this course students will be able to solve related problems.
Course Outline:
Rotation with Constant Angular Acceleration, Relating the Linear and Angular Variables, Kinetic Energy of Rotation, Calculating the Rotational Inertia, Torque, Newton"s Second Law for Rotation, Work and Rotational Kinetic Energy, Rolling as Translation and Rotation Combined, The Kinetic Energy of Rolling, The Forces of Rolling, Angular Momentum, Newton"s Second Law in Angular Form, The Angular Momentum of a System of Particles, The Angular Momentum of a Rigid Body Rotating About a Fixed Axis, Conservation of Angular Momentum, Newton"s Law of Gravitation, Gravitation and the Principle of Superposition, Gravitation Near Earth"s Surface, Gravitation Inside Earth, Gravitational Potential Energy, Planets and Satellites, Kepler"s Laws, Satellites: Orbits and Energy, Einstein and Gravitation, A Closer Look at Heat and Work, The First Law of Thermodynamics, Some Special Cases of the First Law of Thermodynamics, Heat Transfer Mechanisms, Avogadro"s Number, Ideal Gases, Pressure, Temperature, and RMS Speed, Translational Kinetic Energy, Mean Free Path, The Distribution of


Molecular Speeds, The Molar Specific Heats of an Ideal Gas, Degrees of Freedom and Molar Specific Heats, A Hint of Quantum Theory, The Adiabatic Expansion of an Ideal Gas, Irreversible Processes and Entropy, Change in Entropy, The Second Law of Thermodynamics, Entropy in the Real World: Engines, Entropy in the Real World: Refrigerators, The Efficiencies of Real Engines, A Statistical View of Entropy
Text Book: Physics by Hallidy, Resnick and Walker: $8{ }^{\text {th }}$ Edition

## CS-406 Software Packages

## Credit Hours: 3 (2+1)

## Specific Objectives of the Course:

The purpose of this course is to teach students the use of mathematical software's like MATLAB, MAPLE, and MATHEMATICA for solving computationally-difficult problems in mathematics. The students shall become well-versed in using these mathematical software and shall learn a number of techniques that are useful in calculus as well as in other areas of mathematics.

## Course Outline:

The contents of the course are not fixed, however the following points should be kept in mind while teaching the course. The course should be taught in a computer lab setting. Besides learning to use the software, the students must be able to utilize the software to solve the computationally difficult problems in calculus and other areas of mathematics. At the end of the course, the students should have a good command on at least two of the three programs mentioned above. Introduction to simulations by using the mentioned software.

## Recommended Books:

1. D. M. Etter, D, Kuncicky, D. Hull, Introduction to MATLAB, Prentice Hall, Englewood Cliffs,NJ, USA, 2001.
2. F. Garven, The Mapple Book, Chapman \& Hall/CRC, 2002.
3. S. Kaufmann, Mathematica as a Tool, An Introduction with Practical Examples, Springer, New York, 1994.

## MATH-325 Affine Euclidean Geometry

Specific Objectives of course: To familiarize mathematics students with the axiomatic approach to geometry from a logical, historical, and pedagogical point of view and introduce them with the basic concepts of Affine Geometry, Affine spaces and Platonic Polyhedral.

## Course Outline:

Vector spaces and affine geometry: Collinearity of three points, ratio $A B / B C$. Linear combinations and linear dependent set versus affine combinations and affine dependent sets.
Classical theorems in affine geometry: Thales, Menelaus, Ceva, Desargues. Affine subspaces, affine maps. Dimension of a linear subspace and of an affine subspace.
Euclidean geometry: Scalar product,
Cauchy-Schwartz inequality: norm of a vector, distance between two points, angles between two non-zero vectors. Pythagoras theorem, parallelogram law, cosine and sine rules. Elementary geometric loci.
Orthogonal transformations: Isometries of plane (four types), Isometries of space (six types). Orthogonal bases.
Platonic polyhedra: Euler theorem on finite planar graphs. Classification of regular polyhedra in space. Isometries of regular polygons and regular polyhedra.


## Recommended Books:

1. E. Rees, Notes on Geometry, Springer, 2004.
2. M. A. Armstrong, Groups and Symmetry, Springer, 1998.
3. H. Eves, Fundamentals of Modern Elementary Geometry, Jones and Bartlett Publishers International, 1992
S. Stahl, The Poincare Half-Plane A Gateway to Modern Geometry, Jones and Bartlett Publishers International, 1993.

## MATH-347 Partial Differential Equations

## Specific Objectives of the Course:

Partial Differential Equations (PDEs) are at the heart of applied mathematics and many other scientific disciplines. The course aims at developing understanding about fundamental concepts of PDEs theory, identification and classification of their different types, how they arise in applications, and analytical methods for solving them. Special emphasis would be on wave, heat and Laplace equations.

## Course Outline:

First order PDEs: Introduction, formation of PDEs, solutions of PDEs of first order, The Cauchy"s problem for quasilinear first order PDEs, First order nonlinear equations, Special types of first order equations.
Second order PDEs: Basic concepts and definitions, Mathematical problems, Linear operators, Superposition,
Mathematical models: The classical equations, the vibrating string, the vibrating membrane, conduction of heat solids, canonical forms and variable, PDEs of second order in two independent variables with constant and variable coefficients, Cauchy's problem for second order PDEs in two independent variables
Methods of separation of variables: Solutions of elliptic, parabolic and hyperbolic PDEs in Cartesian and cylindrical coordinates Laplace transform: Introduction and properties of Laplace transform, transforms of elementary functions, periodic functions, error function and Dirac delta function, inverse Laplace transform, convolution theorem, solution of PDEs by Laplace transform, Diffusion and wave equations
Fourier transforms: Fourier integral representation, Fourier sine and cosine representation, Fourier transform pair, transform of elementary functions and Dirac delta function, finite Fourier transforms, solutions of heat, wave and Laplace equations by Fourier transforms.

## Recommended Books:

1. Myint UT, Partial Differential Equations for Scientists and Engineers, 3rdedition, North Holland, Amsterdam, 1987.
2. Dennis G. Zill, Michael R. Cullen, Differential equations with boundary value problems, Brooks Cole, 2008.
3. John Polking, Al Boggess, Differential Equations with Boundary Value Problems, 2nd Edition, Pearson,July 28, 2005.
4. J. Wloka, Partial Differential Equations, Cambridge University press, 1987.

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## MATH-348 Real Analysis-II

## Specific Objectives of the Course:

A continuation of real analysis, this course rigorously develops integration theory. Like real integral calculus and real analysis emphasizes on proofs.

## Course Outline:

The Riemann-Stieltjes Integrals: Definition and existence of integrals. Properties of integrals. Fundamental theorem of calculus and its applications. Change of variable theorem. Integration by parts.
Functions of Bounded Variation: Definition and examples. Properties of functions of bounded variation.
Improper Integrals: Types of improper integrals, tests for convergence of improper integrals. Beta and gamma functions. Absolute and conditional convergence of improper integrals. Sequences and Series of Functions: Power series, definition of point-wise and uniform convergence. Uniform convergence and continuity. Uniform convergence.

## Recommended Books:

34. S. Lang, Analysis I, II, Addison-Wesley Publ. Co. Reading, Massachusetts,1968.
35. W. Rudin, Principles of Mathematical Analysis, 3rd Ed., McGraw-Hill, 1976.
36. K. R. Davidson and A. P. Donsig, Real Analysis with Real Applications, Prentice Hall Inc., Upper Saddle River, 2002.
37. G. B. Folland, Real Analysis, 2nd Edition, John Wiley and Sons, New York, 1999.
38. E. Hewitt and K. Stromberg, Real and Abstract Analysis, Springer-Verlag, Berlin Heidelberg New York, 1965.
39. . H. L. Royden, Real Analysis, 3rd Edition, Macmillan, New York, 1988.
40. G. Bartle , R. Sherbert , Introduction to Real Analysis, 3 rd edition, John Wiley, New York, 1999

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## Semester-VII

| Categories | Course Code | Course Title | Marks | Credit Hours |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Allied Course | STAT 345 | Probability and Probability <br> Distribution-II | 100 | 3 |  |  |  |  |  |
| Allied Course | PHYS 103 | Classical Mechanics-I | 100 | 3 |  |  |  |  |  |
| Major | MATH 449 | Numerical Analysis | 100 | 3 |  |  |  |  |  |
| Major | MATH 450 | Functional Analysis | 100 | 3 |  |  |  |  |  |
| Major | MATH | Elective-I | 100 | 3 |  |  |  |  |  |
| Major | MATH 499 | Field Experience/ Internship | 100 | 3 |  |  |  |  |  |
| Teaching of the Holy Quraan with Translation |  |  |  |  |  |  | Non-Credit |  |  |
| Total |  |  |  |  |  |  |  | $\mathbf{6 0 0}$ | $\mathbf{1 8}$ |

## STAT 345 Probability and Probability Distributions-II

## Course Objectives:

- This course is designed to give students a conceptual knowledge of continuous random variables and probability theory.
- This course provides the fundamentals of probability theory in different disciplines.
- This course helps to model the uncertain behavior from the real life scenario.


## Learning Outcomes:

- Understand the basic concepts and applications of probability.
- Investigate the nature of stochastic process and apply suitable probability distributions for the random variable generated from such process.
- Find probabilities using probability distributions.
- Use probability concepts and laws in decision analysis.


## Course Contents:

Continuous Probability Distributions: Properties and Application of Continues Uniform, Normal, Log-Normal, Exponential, Gamma, Beta, Weibull, Cauchy, Laplace, Logistic, Pareto, Rayleigh. Distributions of functions of random variables using Cumulative Dist. Function (CDF), Moment Generating Function (MGF) and Transformation methods, Bivariate Normal distribution.

Sampling Distributions: Chi-square, t and F distributions and their derivations and properties.
Central limit Theorem, Weak and Strong Laws.


Order statistics: Distributions of $\mathrm{r}^{\text {th }}$ and $\mathrm{s}^{\text {th }}$ order statistics.

## Books Recommended

1. Thomas A. Seveini (2005) "Elements of Distribution Theory" Cambridge University Press, Cambridge.
2. Hirai, A.S. (1998), "A Course in Mathematical Statistics", Ilmi Kutab Khana, Lahore.
3. Fridett, B. \& Gray, L. (1997). "A Modern Approach to Probability Theory" Birkhallser, Boston.
4. Freund, J. E. (1997). "Mathematical Statistics", Prentice Hall, New Jersey.
5. Mood, A.M, Graybill, F.A. and Boss, D.C. (1997), "Introduction to the Theory of Statistics", McGraw Hill, New York.
6. Hogg, R.M. and Craig, A.T. (1995), "Introduction to Mathematical Statistics". Prentice Hall, Engle wood Cliffs, New Jersey.

## PHYS-351 Classical Mechanics I

## Cr.Hrs: 3

Prerequisite: Mechanics
Introduction
Newtonian mechanics: Space and Time, Newton's laws, Frame of Reference, The equation of motion for a particle, Conservative Theorems, Energy, Limitations of Newtonian mechanics.
Oscillations: Simple harmonic oscillator, harmonic oscillations in two dimensions, Phase diagrams, damped oscillations, sinusoidal deriving forces, physical systems, principle of superposition.
Some methods in calculus of variations: Statement of the problem, Euler's equation, The second form of the Euler equation, functions with several dependent variables, Euler equations when auxiliary conditions are imposed, the $\delta$ notation.
Hamilton's principle, Lagrangian and Hamiltonian dynamics: Hamilton's principle, applications of Hamilton's principle, generalized coordinates, Lagrange's equations of motions in generalized coordinates, Lagrange's equations with undermined multipliers, Equivalence of Lagrange's and Newton's equations, Essence of Lagrange's dynamics, a theorem concerning the kinetic energy, Conservation theorems revisited, Canonical equations of motion, Hamiltonian dynamics and some comments regarding dynamical variables and variational calculations in physics, phase space and Liouville's theorem, Virial theorem, applications of Virial theorem.
Central force motion: Reduced mass, conservation theorems, first integrals of the motion, equation of motion, orbits in a central field, centrifugal energy and effective potential, Planetray motion and Kepler's problem, orbital dynamics, apsidal angles and precession, stability of circular orbits.

## Recommended Book:

1. S.T. Thornton, J.B. Marion, "Classical Dynamics of Particles and Systems", Brooks

Cole; 5th ed. (2003).
2. T. L. Chow, "Classical Mechanics", John Wiley, (1995).
3. H. Goldstein "Classical Mechanics" latest edition.

35. MATH-449 Numerical Analysis

## Specific Objectives of the Course:

This course is designed to teach the students about numerical methods and their theoretical bases. The students are expected to know computer programming (i.e. Matlab, Mathcad etc.) to be able to write program for each numerical method. Knowledge of calculus and linear algebra would help in learning these methods and to form basic Matlab codes for these.

## Course Outline:

Error analysis: Floating point arithmetic, approximations and errors.
Methods for the solution of nonlinear equations: Bisection method, regula-falsi method, fixed point iteration method, Newton-Raphson method, secant method, error analysis for iterative methods.
Interpolation and polynomial approximation: Lagrange interpolation, Newton"s divided difference formula, forward, backward and centered difference formulae, interpolation with a cubic spline, Hermite interpolation, least squares approximation.
Numerical differentiation: Forward, backward and central difference formulae, Richardson "s extrapolation.
Numerical integration: Rectangular rule, trapezoidal rule, Simpson"s $1 / 3$ and $3 / 8$ rules, Boole"s and Weddle"s rules, Newton-Cotes formulae, Gaussian quadrature.
Numerical solution of a system of linear equations: Direct methods: Gaussian elimination method, Gauss-Jordan method; matrix inversion; LU-factorization; Doolittle"s, Crout"s and Cholesky"s methods,
Iterative methods: Jacobi, Gauss-Seidel and SOR. The use of software packages/programming languages for above mentioned topics is recommended.

## Recommended Books:

1. C.F. Gerald and P.O. Wheatley, Applied Numerical Analysis, Pearson Education, Singapore, 2005.
2. R. L. Burden and J. D. Faires: Numerical Analysis, latest edition, PWS Pub. Co.
3. . J.H. Mathews, Numerical Methods for Mathematics, latest Edition, Prentice Hall International.
4. S. C. Chapra and R. P. Canale: Numerical Methods for Engineers, 6th edition, McGraw Hill.
5. W. E. Boyce, R. C. DiPrima, Elementary Differential Equations and Boundary Value Problems, John Wiley <br>\& Sons, Inc., 2001.
6. L. Debnath, Nonlinear Partial Differential Equations for Scientists and Engineers, Birkhauser-Boston, 2005.
7. Alexander Komech, Andrew Komech, Principles of Partial Differential Equations, Springer-New York, 2009.
8. H. Richard, Elementary Applied Partial Differential Equations, Prentice-Hall International, Inc., London 1987.
9. Weinberger, Hans F., A First Course in Partial Differential Equations with Complex Variables and Transform Methods, Dover Publications, Inc., 1995.
10. R. Kent Nagle, Edward B. Saff, Arthur David Snider, Fundamentals of Differential Equations,


## MATH-450 Functional Analysis

Specific Objectives of course: This course extends methods of linear algebra and analysis to spaces of functions, in which the interaction between algebra and analysis allows powerful methods to be developed. The course will be mathematically sophisticated and will use ideas both from linear algebra and analysis.

## Course Outline:

Metric Space: Review of metric spaces, Convergence in metric spaces, Complete metric spaces, Dense sets and separable spaces, No-where dense sets, Baire category theorem.
Normed Spaces: Normed linear spaces, Banach spaces, Equivalent norms, Linear operator, Finite dimensional normed spaces, Continuous and bounded linear operators, Dual spaces.
Inner Product Spaces: Definition and examples, Orthonormal sets and bases, Annihilators, projections, Linear functionals on Hilbert spaces. Reflexivity of Hilbert spaces.

## Recommended Books:

1. A. V. Balakrishnan, Applied Functional Analysis, 2nd edition, Springer-Verlag, Berlin, 1981.
2. J. B. Conway, A Course in Functional Analysis, 2nd ed., Springer-Verlag, Berlin, 1997.
3. K. Yosida, Functional Analysis, 5th ed., Springer-Verlag, Berlin, 1995.
E. Kreyszig, Introduction to Functional Analysis with Applications, John Wiley and Sons, 2004.

## MATH- Elective-I

## MATH-500 Capstone Project/ Thesis

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## Semester-VIII

| Categories | Course <br> Code | Course Title | Marks | Credit Hours |  |  |  |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Major | MATH 440 | Integral Equation | 100 | 3 |  |  |  |
| Major | MATH 452 | Differential Geometry | 100 | 3 |  |  |  |
| Major | MATH- | Elective-II | 100 | 3 |  |  |  |
| Major | MATH- | Elective-III | 100 | 3 |  |  |  |
| Major | MATH 500 | Capstone Project/ Thesis | 100 | 3 |  |  |  |
| Teaching of the Holy Quraan with Translation |  |  |  | Non-Credit |  |  |  |
| $\mathbf{5}$ Total |  |  |  |  |  | $\mathbf{5 0 0}$ | $\mathbf{1 5}$ |

## MATH-440 Integral Equation

Specific Objectives of course: Many physical problems that are usually solved by differential equation methods can be solved more effectively by integral equation methods. This course will help students gain insight into the application of advanced mathematics and guide them through derivation of appropriate integral equations governing the behavior of several standard physical problems.
Course Outline: Linear integral equations of the first kind, Linear integral equations of the second kind. Relationship between differential equation and Volterra integral equation. Neumann series. Fredholm Integral equation of the second kind with separable Kernels. Eigenvalues and eigenvectors. Iterated functions. Quadrature methods. Least square methods. Homogeneous integral equations of the second kind. Fredholm integral equations of the first kind. Fredholm integral equations of the second kind. Abel"s integral equations. Hilbert Schmidt theory of integral equations with symmetric Kernels. Regularization and filtering techniques.

## 1. Recommended Books:

1. C. T. H. Baker, Integral Equations, Clarendon Press, 1977.
2. F. Smithies, Integral Equations, Cambridge University Press, 1989.
3. A. M. Wazwaz, A first Course in Integral Equations, World Scientific Pub., 1989.
4. W. V. Lovitt, Linear Integral Equations, Dover Publications, 2005.
5. MATH-452 Differential Geometry

## MATH-452 Differential Geometry

Credit Hours: $3+0$

## Specific Objectives of the Course:

After having completed this course, the students would be expected to understand classical concepts in the local theory of curves and surfaces including normal, principal, mean, curvature, and geodesics. They will also learn about tensors of different ranks.



## Course Outline:

Theory of Space Curves: Introduction, index notation and summation convention. Space curves, arc length, tangent, normal and binormal. Osculating, normal and rectifying planes. Curvature and torsion. The Frenet-Serret theorem. Natural equation of a curve. Involutes and evolutes, helices. Fundamental existence theorem of space curves.
Theory of Surfaces: Coordinate transformation. Tangent plane and surface normal. The first fundamental form and the metric tensor. The second fundamental form. Principal, Gaussian, mean, geodesic and normal curvatures. Gauss and Weingarten equations. Gauss and Codazzi equations.
Tensor Analysis: Einstein summation convention. Tensors of different ranks. Contravariant, covariant and mixed tensors. Addition, subtraction, inner and outer products of tensors.
Contraction theorem, quotient law. The line element and metric tensor. Christoffel symbols.

## Recommended Books:

3. R. S. Millman and G. D. Parker, Elements of Differential Geometry, Prentice-Hall, New Jersey, 1977.
4. A. Goetz, Introduction to Differential Geometry, Addison- Wesley, 1970.
5. E. Kreyzig, Differential Geometry, Dover, 1991.
6. M. M. Lipschutz, Schaum"s Outline of Differential Geometry, McGraw Hill, 1969.
7. D. Somasundaram, Differential Geometry, Narosa Publishing House, New Delhi. 2005.
8. M. R. Spiegel, Vector Analysis, McGraw Hill Book Company, Singapore, 1981.
9. A. W. Joshi, Matrices and Tensors in Physics, Wiley Eastern Limited, 1991.
10. F. Chorlton, Vector and Tensor Methods, Ellis Horwood Publisher, U.K., 1977.

## MATH-225 Affine Euclidean Geometry

## Credit Hours:3+0

Specific Objectives of course: To familiarize mathematics students with the axiomatic approach to geometry from a logical, historical, and pedagogical point of view and introduce them with the basic concepts of Affine Geometry, Affine spaces and Platonic Polyhedral.

## Course Outline:

Vector spaces and affine geometry: Collinearity of three points, ratio AB/BC. Linear combinations and linear dependent set versus affine combinations and affine dependent sets.
Classical theorems in affine geometry: Thales, Menelaus, Ceva, Desargues. Affine subspaces, affine maps. Dimension of a linear subspace and of an affine subspace.
Euclidean geometry: Scalar product,
Cauchy-Schwartz inequality: norm of a vector, distance between two points, angles between two non-zero vectors. Pythagoras theorem, parallelogram law, cosine and sine rules. Elementary geometric loci.
Orthogonal transformations: Isometries of plane (four types), Isometries of space (six types). Orthogonal bases.
Platonic polyhedra: Euler theorem on finite planar graphs. Classification of regular polyhedra in space. Isometries of regular polygons and regular polyhedra.

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## Recommended Books:

1. E. Rees, Notes on Geometry, Springer, 2004.
2. M. A. Armstrong, Groups and Symmetry, Springer, 1998.
3. H. Eves, Fundamentals of Modern Elementary Geometry, Jones and Bartlett Publishers International, 1992
4. S. Stahl, The Poincare Half-Plane A Gateway to Modern Geometry, Jones and Bartlett Publishers International, 1993.


# Elective Courses 

## MATH-461 Mathematical Modeling

Credit Hours: $3+0$

## Specific Objectives of the Course:

Mathematics is used in many areas such as engineering, ecological systems, biological systems, financial systems, economics, etc. In all such applications one approximates the actual situation by an idealized model. This is an introductory course of modeling, consisting of three parts: modeling with ordinary differential equations and their systems; partial differential equations; and integral equations. The course will not be concerned with the techniques for solving the equations but with setting up the equations in specific applications. Whereas the first two types of equations have already been dealt with, the third type has not. Consequently, solutions of the former will be discussed but of the latter will barely be touched upon.

## Course Outline:

Concepts of model, modeling and simulation functions, linear equations, linear-differential equations, nonlinear differential equations and integral equations as models, introduction to simulation techniques ordinary differential equations: modeling with first order differential equations: Newton"s law of cooling; radioactive decay; motion in a gravitational field; population growth; mixing problem; Newtonian mechanics. Modeling with second order differential equations: vibrations; Modeling with periodic or impulse forcing functions, Modeling with systems of first order differential equations; Partial Differential Equations: Methodology of mathematical modeling; objective, background, approximation and idealization, model validation, compounding, Modeling wave phenomena (wave equation); Modeling the heat equation and some application to heat conduction problems in rods, Modeling the potential equation (Laplace equation), Applications in fluid mechanics, gravitational problems, Equation of Continuity.

## Recommended Books:

1. F. R. Giordano, MD.Weir, Differential Equations: A Modeling Approach, AddisonWesley, Reading, Ma, USA, 1994.
2. U. T. Myint, L. Debnath, Partial Differential Equations for Scientists and Engineers (3 ${ }^{\text {rd }}$ edition), North Holland, Amsterdam, 1987.
3. S. Robert, An Introduction to Programming and Numerical Methods in MATLAB, 1987.

## MATH-462 Advanced Group Theory

## Credit Hours: 3+0

Specific Objectives of the Course: The advance algebra is the extention of Abstract algebra which has many applications in structure analysis, etc.
Course Contents: Sylow theory, applications of Sylow theory, generating systems for finite symmetric and alternating groups. Simple groups, simplicity of An for 5, Zassenhaus lemma, Normal series, Composition series, Jordan Holder theorem, Solvable groups, The derived series of a group, The lower and upper Central series of a group and Nilpotent groups.


## Recommended Books:

1. E. Arnold, Rings, Fields and Groups: An Introduction to Abstract Algebra, 1983.
2. A Majeed, Group Theory, Ilmi kitab Khana, 1996.
3. Zia-Ul-Haq, Mathematical Techniques, Carvan Books Publishing Company, 2000.
4. J. B. Farleigh, A First Course in Abstract Algebra (7 $7^{\text {th }}$ edition), Addison-Wesley, Reading, Ma., USA, 2000.
5. I. D. Macdonald, The Theory of Groups, Oxford Clarendon Press, Ma., USA, 1975.
6. K. H. Dar, Abstract Algebra, Ilmi Kitab Khana Lahore, 2001.

## MATH-463 Optimization Theory

## Credit Hours: $3+0$

## Specific Objectives of the Course:

The main objective is to teach the basic notions and results of mathematical programming and optimization. The focus will be to understand the concept of optimality conditions and the construction of solutions. Students should have a good background in analysis, linear algebra and differential equations.

## Course Outline:

Linear programming: simplex method, duality theory, dual and primal-dual simplex methods, Unconstrained optimization: optimality conditions, one-dimensional problems, multidimensional problems and the method of steepest descent. Constrained optimization with equality constraints: optimality conditions, Lagrange multipliers, Hessians and bordered Hessians. Inequality constraints and the Kuhn-Tucker Theorem, The calculus of variations, the Euler-Lagrange equations, functional depending on several variables, variational problems in parametric form, transportation models and networks.

## Recommended Books:

1. L. Elsgolts, Differential Equations and the Calculus of Variations, Mir Publishers, Moscow, 1970.
2. B. S. Gotfried, J. Weisman, Introduction to Optimization Theory, Prentice Hall, Englewood Cliffs, NJ, USA, 1973.
3. D. G. Luenberger, Introduction to Linear and Non-Linear Programming, AddisionWesley, Reading, Ma, USA, 1973.

## MATH-464 Measure Theory

Credit Hours: 3+0
Specific Objective of the Course:
This course is devoted to Lebesgue integration and related topics, a basic part of modern analysis. There are classical and abstract approaches to the integral, and we have chosen the classical one. The classical approach is based on the theory of measure. Measure can be defined and studied in various spaces, but we will primarily consider n-dimensional Euclidean spaces.

## Course Outlines:

Limit superior , Limit inferior, Measure, Outer measure, Lebesgue measure, Counting Measure,



Lebesgue Measurable set, Measurable functions, Elementary properties of measurable function, Lebesgue integral, Riemann integral, Relationship between Riemann and Lebesgue integral, Properties of the Lebesgue integral, The integral of arbitrary measurable functions, Relation between Riemann-Stieltjes and Lebesgue integrals, $L^{p}$ spaces, Properties of $L^{p}$ spaces, Holder inequality, Minkowski inequality.

## Recommended Books:

1. Richard L. Wheeden and Antoni Zygmund, Measure and Integral, An Introduction to Real Analysis, 2015.
2. Elias M. Stein \& Rami Shakarchi, Real Analysis Measure Theory, Integration and Hilbert Spaces, Princeton University Press Princeton and Oxford, 2000.
3. N. L. Carothers, Real Analysis, Cambridge University press, 1999.

## MATH-465 Fluid Mechanics

Credit Hours: $3+0$

## Course Outline:

Real and ideal fluids, Force, Pressure, Density, Specific volume, Specific weight, Stress and strain, Young"s modulus, Viscosity, Surface tension, Steady and unsteady flow, turbulent flow, laminar flow, two-dimensional flow, three-dimensional flow, Eulerian and Lagrangian Flow Descriptions, Path line, Streamline, stream tube, Stream filament, Stream surface, Streak line, The equation of continuity, The acceleration field, The Euler equation, The total derivative, Bernoullies theorem, Flow of dry water continued, Flux, Vorticity and rotation, The velocity potential, Laplace"s equation, Uniform flow, Source and sink, Viscosity, Deformation, The equations of motion for viscous (wet) fluids, The Navier-Stokes equation, Viscous, incompressible, laminar flow, A. channel flow (2D counterpart of pipe flow), No-Slip Condition, Channel flow, Laminar flow in a pipe, Viscous flow past a circular cylinder, Reynolds number, Reynolds number.

## Recommended Books:

1. Buffler, Introduction to fluid mechanics, Department of Physics, University of Cape Town, 2009.
2. Kundu and Cohen, Fluid Mechanics, 4th Edition, by Academic Press, NY. 2008.
3. G. K. Batchelor, An Introduction to Fluid Dynamics, 2nd Edition, by Cambridge University Press, Cambridge. 2000.
4. F. M. White, Fluid Mechanics, 7th Edition, McGraw Hill, NY, 2011.

## MATH-466 Stochastic Processes

Credit Hours: 3+ 0
Specific Objectives of the Course:
The objectives of this course is to make certain that each student knows the theoretical methods of probability models and stochastic processes including Markov chains, Brownian Motion, Queuing theory, and stochastic differential equations.
Course Outline:
Review of probability theory with main emphasis on conditional probability and conditional


expectation, Theory of Markov chains, Continuous-time Markov chains, Renewal theory and its application, Queuing theory, stochastic processes, stopping times, continuous times martingales, the Doob-Meyer Decomposition theorem, continuous square-integrable Martingales, Random Walk, Brownian motion, the strong Markov property and the reflection principal, Brownian Filtration, the Brownian sample path, stochastic integrals, The Ito rule, The Girsanov"s Theorem, stochastic differential equations, strong solutions, weak solutions, Gauss-Markov processes, the general one dimensional linear equation, connections with partial differential equations.

## Recommended Books:

1. H. Taylor and S. Karlin, An Introduction to Stochastic Modeling, $3^{\text {rd }}$ edition, 1998.
2. Sheldon M. Ross, Introduction to Probability Models, $10^{\text {th }}$ edition, 2010.
3. N. Shiryaev, Probability, Springer, New York, 1995.
4. Karatzas, St. Shreve, Brownian Motion and Stochastic Calculus, Springer-Verlag, New York 1992.

## PHYS-467 Quantum Mechanics

## Credit Hours: 3+0

Course Outline:
Wave-Particle, Plan-Einstein relation, Debroglie relations, Schrodinger equation, Normalization of wave function, Waves-Pocket, Heisenberg indeterminacy or UIXCER Taint principal, Phase velocity, Group velocity, Stationary states, Properties of a waves function, Linear operators, Orthogonal basis in waves equation, Closure relation, Parseval relation, Orthonormalization relation, Delta functions, Ketand Bro vectors, the adjoint operators, Eigen value equations and observables, Projection operation, Basic postulates of Quantum theory, Implementations of the Schrödinger Equations, Conservative system, Angular momentum, Time-Energy Uncertainty, Raising operators, Spin Observable, Hormonic oscillator, Hydrogenic atoms, Pauli Exclusion Principal.

## Recommended Books:

1. D. J. Griffiths, Introduction to Quantum Mechanics, latest edition, 1985.
2. R. L. Liboff, Introductory Quantum Mechanics, $4^{\text {th }}$ edition, 1990.
3. Seymour Lipschutz, et.al, Quantum Mechanics, Schaum Outline series, 2000.

## MATH-468 Special Relativity

## Credit Hours: 3+0

## Course Outline:

Introduction to Einsteincs Theory of Special Relativity, Lorentz transformations (one dimensional), length contraction, time dilation and simultaneity, Covariant and contravariant tensors and Einstein Summation convention, velocity addition formulae, 3- dimensional Lorentz transformations, introduction to 4 -vector formalism, Lorentz transformations in the 4 -vector formalism, the Lorentz and Poincare groups, introduction to classical mechanics, Minkowski space time and null cone, 4-velocity, 4-momentum and 4-force, application of special relativity to Doppler shift and Compton effect, particle scattering, binding energy, particle production and


decay, electromagnetism in relativity, electric current, Maxwell"s equations and electromagnetic waves, the 4 -vector formulation of Maxwelles equations, special relativity with small acceleration. Review of analytic geometry in three dimension, Tensors and differential geometry, Isometries and Killing equations, Einstein"s theory of relativity, Schwarzschild solution, Gravitational deflection of light, Field theory, Black holes, Relativistic cosmology.

## Recommended Books

1. H. Golstein, Classical Mechanics, Addison Wesley, 1962.
2. J. B. Kogut, Introduction to Relativity, Wiley, 2011.
3. H. N. Ohanian, Special relativity: A Modern Introduction, 2010.
4. R. D. Inverno, Introducing Einstein's Relativity, Oxford University Press, 1992.
5. S. W. Hawking and G. F. R. Ellis, The Large Scale Structure of Space-time, 2008.

## MATH-469 Heat and Mass Transfer

Credit Hours: 3+0

## Specific Objectives of the Course:

This course will provide the basic concepts of conduction, convection and radiation heat transfer. It will help the students to understand how to formulate and be able to solve one and two dimensional conduction heat transfer problems. Solution techniques will include both closed form and numerical methods. Convection effects will be included as boundary conditions. Moreover, the students will understand the fundamentals of the relationship between fluid flow, convection heat transfer and mass transfer and will apply empirical correlations for both forced and free convection to determine values for the convection heat transfer coefficient. They will then calculate heat transfer rates using the coefficients. In addition to this, students will understand the basic concepts of radiation heat transfer to include both black body radiation and gray body radiation and will be able to evaluate radiation view factors using tables and the view factor relationships.
Course Outline:
One-dimensional heat conduction problem, Two-dimensional heat conduction problem, Transient heat conduction, Principles of convection heat and mass transfer, Equations of forced convections, Equations of free convections, Principles of radiation heat transfer, Radiation exchange between surfaces, Heat exchanger analysis, Mass transfer.

## Recommended Books:

1. F. Incropera, D. DeWitt, Fundamentals of Heat Mass Transfer, 6th edition, 2011.

## MATH-470 Advanced Number Theory

## Credit Hours: 3+0

Specific Objective of the Course:
This course contains some advance topics of number theory, this course enable the students to solve higher degree congruence"s. In this course the students also learn to solve an equation containing three variables using modulo concepts etc. This course also familiarize the students with the solutions of an equation in $Z_{n}$ where n is prime or composite. This subject covers some


topics of graduate level.
Course Outline:
Preliminaries: Well-ordering principle. Principle of finite induction.
Divisibility theory: The division algorithms. Basis representation theorem. Prime and composite numbers. Canonical decomposition. The greatest common divisor. The Euclidean algorithm. The fundamental theorem of arithmetic. Least common multiple. Linear Diophantine equations: Congruences. Linear congruences. System of linear congruences. The Chinese remainder theorem. Divisibility tests. Solving polynomial congruences. Fermat's and Euler's theorems. Wilson's theorem.
Arithmetic functions: Euler's phi-function. The functions of $\mathbf{J}$ and sigma. The Mobius function. The sieve of Eratosthenes. Perfect numbers. Fermat and Mersenne primes.
Primitive Roots and Indices: The order of an integer mod n . Primitive roots for primes. Composite numbers having primitive roots.
Quadratic residues: Legendre symbols and its properties. The quadratic reciprocity law. Quadratic congruences with composite moduli. Pythagorean triples. Representing numbers as sum of two squares.

## Recommended Books:

1. D.M. Burton, Elementary Number Theory, McGraw-Hill, 2007.
2. W.J. Leveque, Topics in Number Theory, vols. I and II, Addison- Wesley, 1956.
3. S.B. Malik, Basic Number Theory, Vikas Publishing house, 1995.
4. K.H. Rosen, Elementary Number Theory and its Applications, 5th edition, AddisonWesley, 2005.
5. I. Niven, H.S. Zuckerman, H.L. Montgomery, An Introduction to the theory of Numbers, John Wiley and Sons, 1991.
6. . A. Adler, J.E. Coury, The Theory of Numbers, Jones and Bartlett Publishers, 1995.

## MATH-471 Analytical Dynamics

Credit Hours: $3+0$

## Course Outline:

Dynamics of a Rigid Body; Moments and product of inertia, D' Alembert's principle, Motion about a fixed axis, Linear Momentum and Kinetic energy of a rigid body, Compound pendulum, Motion in two dimension, Finite forces; impulsive forces, Lagrange's equations in generalized coordinates, Dynamics of a Particle; Uniplanar motion, acceleration parallel to fixed axes, polar coordinates, moving axes, central forces, stability of orbits, acceleration varying as the inverse square of the distance, Kapler's laws, Planetary motions, Tangential and Normal accelerations, Motion in a resisting medium, Angular momentum and rate of change of angular momentum for a system of particles.

## Recommended Books:

1. S. L. Loney, Dynamics of a particle and Rigid Bodies, 2018
2. F. Charlton, A Text Book of Dynamics, 1983.

## MATH-472 Difference Equations



## Credit Hours: 3+0

Aims and objectives: Many problems in probability give rise to difference equations. Difference equations relate to differential equations as discrete mathematics relates to continuous mathematics. Anyone who has made a study of differential equations will know that even
supposedly elementary examples can be hard to solve. By contrast, elementary difference equations are relatively easy to deal with. Aside from probability, computer scientists take an interest in difference equations for a number of reasons. For example, difference equations frequently arise when determining the cost of an algorithm in big-O notation. Since difference equations are readily handled by program, a standard approach to solving a nasty differential equation is to convert it to an approximately equivalent difference equation.
Course Contents: Introduction to difference equations, classification of difference equations, homogeneous and inhomogeneous, linear and nonlinear difference equations of order one, two and n-order. Difference equations with variable and constants coefficients. Solutions of difference equations by undermined coefficient method. Characteristic polynomial of the equation. Complementary and particular solutions of difference equations. Some Applications of difference equations to real world problems.

## Recommended Books:

1. S.M. Yusuf, et.al, Mathematical Methods, Ilmi Kitab Khana, Lahore, 1998.
2. Chiang, Alpha. Fundamental Methods of Mathematical,Economics, McGraw-Hill, $3^{\text {rd }}$ Ed 2007
3. Baumol, William. Economic Dynamics, Macmillan, third edition, 1970

## MATH-473 Convex Analysis

## Credit Hours: 3+0

Specific Objectives of the Course: This course will focus on fundamental subjects in convexity, duality, and convex optimization algorithms. The aim is to develop the core analytical and algorithmic issues of continuous optimization, duality, and saddle point theory using a handful of unifying principles that can be easily visualized and readily understood.
Course Contents: Basic convexity concepts, Definition of a Convex Set, Examples of Convex Sets, Convex Cones Supporting Hyperplane, Hyperplanes and conjugacy, Separation of Disjoint Convex Sets, Convex Functions, Definition of a Convex Function, Properties of Convex Functions, Convexity of Level Sets, Continuity of Convex Functions, Generalizations of Convex Functions, Quasiconvex Functions, Pseudo convex Functions, Relationship Among Various Types of Convexity, Convexity at a Point, Convexity and optimization, Polyhedral convexity, Lagrangian duality, Fenchel duality, conic duality, saddle point theory. Convexity in Hilbert spaces and its applications.

## Recommended Books:

1. Bertsekas, Dimitri. Convex Optimization Theory. Athena Scientific, 2009
2. Rockafellar, Ralph. Convex Analysis. Princeton University Press, 1996.
3. Boyd, Stephen, and Lieven Vandenberghe. Convex Optimization. Cambridge University Press, 2004
4. Kreyszig - Introductory Functional Analysis with Applications, latest edition, 2001.

## ECON-474Econometrics




Credit Hours: 3+0

## Specific Objectives of Course:

The course provides a foundation to estimate econometric models with special emphasis on ordinary least square method.

## Course Outline:

Introduction, definition and scope of econometrics, econometric models vs. statistical models, ingredients of econometric modeling, specification, estimation, verification or evaluation and forecasting, The classical linear regression model, the simple linear regression model (SLRM), estimation of SLRM by ordinary least squares (OLS) interpretation of estimated coefficients and their economic meanings, hypothesis testing and analysis of variance, the multiple linear regression model (MLRM), estimation of MLR model by OLS and its assumptions interpretation of estimated coefficients and their economic meanings, regression through origin, double log estimation and computation of elasticities, using R-square and adjusted R-square as a measure of „Goodness of Fit" and some problems with its use, testing the significance of individual coefficients, testing the significance of the model as a whole, analysis of variance.
Recommended Books:

1. D. Gujrati, Basic Econometrics, Mc-Graw Hill, (latest edition), 2011.
2. Koutsoyiannis, Theory of Econometrics, McMillan, (latest edition), 2010
3. G. M. K Madnani, Introduction to Econometrics Principles and Applications, 2008
4. R.J. Wonnacot, Econometrics, John Wiley, New York, 2005.

## MATH-475 Lie Algebra

Credit Hours: $3+0$
Specific Objectives of the Course: This course focus on Lie algebra which In mathematics, a Lie algebra is a vector space together with a non-associative operation called the Lie bracket, an alternating bilinear map, satisfying the Jacobi identity.

Course Outlines: Introduction to Lie algebra, history definitions. Generators and dimension, derivation of Lie algebra. Matrix Lie algebras, examples and split of Lie algebras. Associative algebras with commutator brackets. Sub algebras, ideals and homomorphism. Direct sum and semi direct product. Classification in Lie algebra.

## Recommended Books:

1. B.Nicolas Lie Groups and Lie Algebras: Chapters 1-3. Berlin•Heidelberg•New York: Springer, 1989.
2. Hall, Brian C. Lie groups, Lie algebras, and Representations: An Elementary Introduction. Graduate Texts in Mathematics. 222 (2nd ed.). Springer, 2015.
3. Hofmann, Karl H.; Morris, Sidney A , The Lie Theory of Connected Pro-Lie Groups. European Mathematical Society. 2007.

## MATH-476 Galois Theory

Credit Hours: $3+0$
Specific Objectives of the Course: In mathematics, Galois theory provides a connection between field theory and group theory. Using Galois theory, certain problems in field theory can be reduced to group theory, which is in some sense simpler and better understood.



Course Outlines: Introduction to Galios theory, pre-history and application to classical problems.Permutation group approach to Galios theory. Soluble groups and solutions by radicals. Inverse Galios problems.

## Recommended Books:

1. Edwards, Harold M. Galois Theory. Springer-Verlag. 1984. ISBN 0-387-90980-X.
2. Postnikov, M. M. Foundations of Galois Theory. Dover Publications, 2004, ISBN 0-486-43518-0.
3. Rotman, Joseph, Galois Theory (2nd ed.). Springer, 1998, ISBN 0-387-98541-7.

## MATH-477 Simulations

Credit Hours: $3+0$
Specific Objectives of the Course: A simulation is an approximate imitation of the operation of a process or system; that represents its operation over time. Simulation is used in many contexts, such as simulation of technology for performance optimization, safety engineering, testing, training, education, and video games.
Course Outlines: Introduction to simulations, classifications and terminology. Computer simulations. Applications of Simulations in training, economics and health care. Tools for simulations. Simulations of basic mathematical models in physics and biology.

## Recommended Books:

1. J. A. Sokolowski, Banks, C.M. (2009). Principles of Modeling and Simulation. John Wiley \& Son. p. 6. ISBN 978-0-470-28943-3.
2. Zeigler, B. P., Praehofer, H., \& Kim, T. G. (2000) "Theory of Modeling and Simulation: Integrating Discrete Event and Continuous Complex Dynamic Systems", Elsevier, Amsterdam.
3. Kuhl, F., Weatherly, R., \& Dahmann, J. (1999). Creating computer simulation systems: an introduction to the high-level architecture. Prentice Hall PTR.

## MATH-478 Dynamical Systems

Credit Hours: $3+0$
Specific Objectives of the Course: To understand by the graduates about dynamical systems. The said systems in which a function describe the time dependence of a point in a geometrical space. Examples include the mathematical models that describe the swinging of a clock pendulum, the flow of water in a pipe, and the number of fish each springtime in a lake.
Course Outlines: Introduction to dynamical systems and history. Linear and nonlinear dynamical systems, Examples. Abstract dynamical systems, Complex dynamical systems. Local dynamics, Ergodic systems and bifurcation analysis. Methods of dynamics, Applications of dynamical systems. Stability of dynamical systems.

## Recommended Books:

1. Strogatz, S. H. Nonlinear Dynamics and Chaos: with Applications to Physics, Biology


and Chemistry. Perseus, 2001.
2. Katok, A.; Hasselblatt, B. Introduction to the Modern Theory of Dynamical Systems. Cambridge: Cambridge University Press. ISBN 978-0-521-34187-5, 1995.
3. T.Jackson, A. Radunskaya, Applications of Dynamical Systems in Biology and Medicine. Springer, 2015.
4. Q. K. Ghori, Introduction to Mechnics, Ilmi Kitabkhana Lahore, Revised Edition, 2003.

## MATH-479 Mathematical Physics

## Credit Hours: $3+0$

Specific Objectives of the Course: Mathematical physics refers to the development of mathematical methods for application to problems in physics.
Course Outlines: Introduction and history of mathematical physics. Composition of forces, Center of mass and gravity, Theorems of pappus, Friction, Kinematics, Rectilinear motions and motion of projectiles.

## Recommended Books:

1. Edwin Bidwell Wilson, Vector Analysis: A text-book for the use of students of mathematics and physics, 1901.
2. Mathematics With Mathematical Physics Bsc, London"s Global University,2019.
3. Dr. S.L. Kakani, Mathematical Physics, CBS Publishers \& Distributors; 2nd edition 2004.
4. Q. K. Ghori, Introduction to Mechanics, Ilmi Kitabkhana Lahore, Revised Edition, 2003.

## MATH-480 History of Mathematics

Credit Hours: $3+0$
Specific Objectives of the Course: To train the students to know how and why mathematics came into being.
Course Outlines: Egyptian geometry and arithmetical operations, Babylonian geometry and number system, Euclidecs Elements, The Geometry of Archimedes and Eratosthenes, Ethno mathematics, Hippocrates" quadrature of the lune, Euclid"s proof of the Pythagorean theorem, Euclid and the infinitude of primes, Archimedes" determination of circular area , Heron"s formula for triangular area, Chinese mathematics and problem solving, Islamic mathematics and art, Al-Khwarizmi on quadratic equations, Alberuni on theory of functions, Medieval mathematics, especially solving the cubic, The Copernican revolution, Newton and Leibniz and the calculus, Eulerian contributions. Gauss, Cauchy, Riemann and rigorous calculus/analysis, Fibonacci and the rabbit problem, Cardano and the solution of the cubic, Napier and logarithms, Newton"s binomial theorem ,Newton \& Leibniz on the calculus, Euler on number theory , Cantor and the infinite. Banach and Mittage-Leffler contributions.

## Recommended Books:

1. Katz, Victor J., A History of Mathematics: An Introduction, 3rd ed., Addison-Wesley, 2009.
2. Dunham, William, Journey Through Genius: The Great Theorems of Mathematics, Penguin, 1990.
3. Burton, David M., The History of Mathematics: An Introduction, 7th ed., McGraw-Hill,


4. 
5. Boyer, Carl B. and (revised by) Uta Merzbach, A History of Mathematics, New York: John Wiley, 2 nd ed., 1989.
6. Boyer, Carl B. and (revised by) Uta Merzbach, A History of Mathematics, New York: John Wiley, 2 nd ed., 1989.
7. Levine, Gavrielle, Prior Mathematics History, Anticipated Mathematics Teaching Style, and Anxiety for Teaching Mathematics among Pre-Service Elementary School Teachers, 1993

## MATH-481 Mathematical Biology

Credit Hours: $3+0$

## Specific Objectives of the Course:

Interactions between the mathematical and biological sciences have been increasing rapidly in recent years. This course will help the students to understand how to form mathematical models from bio data or infectious disease.
Course Outlines: History of mathematical models. Formation of biological model. Equilibria and global and local stability of biological models. Investigations of some basic biological model including SI, SIS, SIR epidemic and endemic disease model, Logistic equation and evolution models, the Lotka Voltera models. Introduction to optimal control and some important applications.

## Recommended Books:

1. Jeffrey R. Chasnov , Mathematical Biology, Hong Kong University Press, 2009
2. Elizabeth S. Allman and John A. Rhodes, An Introduction to Mathematical Models in Biology, Cambridge University Press, 2004.
3. Miklos Farkas , Dynamical Models in Biology, Academic Press, New York, 2001.

## MATH-482 Lie Group

Credit Hours: $3+0$
Specific Objectives of the Course: To understand concepts a Lie group is a group that is also a differentiable manifold, with the property that the group operations are smooth. Lie groups are named after Norwegian mathematician Sophus Lie, who laid the foundations of the theory of continuous transformation groups.
Course Outlines: Introduction to Lie group and history. Definitions and examples of Lie groups, subgroups. Classification of Lie groups and matrix Lie group. Homomorphism and isomorphism in Lie groups. Simple Lie group and its connections. Topological Lie group and examples.

## Recommended Books:

1. Hall, Brian C. Lie Groups, Lie Algebras, and Representations: An Elementary Introduction, Graduate Texts in Mathematics, 222 (2nd ed.), Springer, 2015.
2. Sattinger, David H.; Weaver, O. L. Lie groups and algebras with applications to physics, geometry, and mechanics. Springer-Verlag, 1986.
3. Serre, Jean-Pierre Lie Algebras and Lie Groups: Lectures given at Harvard University, Lecture notes in mathematics, Springer, 2000.



## MATH-483 Rings and Modules

Credit Hours: $3+0$
Specific Objectives of the Course:
In mathematics, a module is one of the fundamental algebraic structures used in abstract algebra. A module over a ring is a generalization of the notion of vector space over a field, wherein the corresponding scalars are the elements of an arbitrary given ring (with identity) and a multiplication (on the left and/or on the right) is defined between elements of the ring and elements of the module. A module taking its scalars from a ring $R$ is called an $R$-module.
Course Outlines: Introduction and definitions of rings and modules, examples. Rings and ideals, radicals, Chinese Remainder Theorem. Modules over rings, Polylinear constructions and examples. Noetherian modules and Unique factorization domains. Polynomial rings over unique factorization domains. Irreducibility criteria of rings and modules, examples. Classifications of modules.

## Recommended Books:

1. F.W. Anderson and K.R. Fuller: Rings and Categories of Modules, Graduate Texts in Mathematics, Vol. 13, 2nd Ed., Springer-Verlag, New York, 1992.
2. Dummit, David S. \& Foote, Richard M, Abstract Algebra. Hoboken, NJ: John Wiley \& Sons, 2004.
3. Lambek, Joachim. Lectures on rings and modules. Vol. 283. American Mathematical Soc. 2009.

## MATH-484 Projective Geometry

Credit Hours: $3+0$
Specific Objectives of the Course: Projective geometry is a branch in mathematics. It is the study of geometric properties that are invariant with respect to projective transformations. This means that, compared to elementary geometry, projective geometry has a different setting, projective space, and a selective set of basic geometric concepts.
Course Outlines: Introduction to Projective geometry and history. Foundation of projective geometry. Synthetic and analytic geometry. Construction of projective space. Projective geometry and homogeneous coordinates. Fundamental requirements of projective geometry. Essential Concepts of Projective Geomtry and Applications of projective geometry. Synthetic projective geometry. Multidimensional projective geometry. Plane projective geometry.

## Recommended Books:

1. L. Blum, F. Cucker, M. Shub and S. Smale. Complexity and Real Computation. SpringerVerlag, New York-etc., 1997
2. W. V. D. Hodge and D. E. Pedoe. Methods of Algebraic Geometry, Volumes 1-3 (Cambridge Mathematical Library). Cambridge Univ. Press, New York, 1994.11
3. R. Baer. Linear Algebra and Projective Geometry. Dover, New York, 2005.


4. E. C. Wallace and S. F. West. Roads to Geometry (3rd Ed.). Prentice-Hall, Upper Saddle River, NJ, 2003.

## Study Tour of Students

Study tours can vary from short term (3 days) to long term (01 week). The study tours have many positive outcomes one of it is a learning component. The students shall develop their trend of research and academics. Therefore, in $8^{\text {th }}$ semester of BS-Mathematics students shall be allowed a study tour by the University of Malakand.

## Note:

Courses included in the General Education Category are designed by the respective departments including their course codes, credit hours and titles (reflected in the scheme of studies). All such courses approved by the Syndicate are available on the university website (www.uom.edu.pk). For any query the office of the Registrar Academics may be approached for clarification/guidance.


